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using Agent Based Model

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Investigation of Frequent Batch Auctions using Agent Based Model *

Takanobu Mizuta [†], Kisyoshi Izumi [‡]

December 8, 2016

Abstract

Recently, the speed of order matching systems on financial exchanges increased due to competition between markets and due to large investor demands. There is an opinion that this increase is good for liquidity by increasing providing liquidity of market maker strategies (MM), on the other hand, there is also the opposite opinion that this speed causes socially wasteful arms race for speed and these costs are passed to other investors as execution costs.

A frequent batch auction (FBA) which reduces the value of speed advantages proposed, however, is also criticized that MM providing liquidity are exposed to more risks, and then they can continue to provide liquidity, then many MM retire, and finally liquidity will be reduced.

In this study we implemented a price mechanism that is changeable between a comparable continuance double auction (CDA) and FBA continuously, and analyzing profits/losses and risks of MM, we investigated whether MM can continue to provide liquidity even on FBA by using an artificial market model.

Our simulation results showed that on FBA execution rates of MM becomes smaller and this causes to reduce liquidity supply by MM. They also suggested that on FBA MM cannot avoid both an overnight risk and a price variation risk intraday, furthermore, it is very difficult that MM is rewarded for risks and continues to provide liquidity. Only on CDA MM is rewarded for risks and continue to provide liquidity.

This suggestion implies that MM that can provide liquidity on CDA cannot continue to provide liquidity on FBA and then many MM retire, finally liquidity will be reduced.

* Note that the opinions contained herein are solely those of the authors and do not necessarily reflect those of Japan Exchange Group, Inc., its subsidiaries, affiliates, and SPARX Asset Management Co., Ltd. This research was partially supported by CREST, JST and JSPS KAKENHI Grant Number 15H02745. Contact: Takanobu Mizuta (mizutata@gmail.com)

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1 Introduction

From 2000's to early 2010's, the speed of order matching systems on financial exchanges increased due to competition between markets and due to large investor demands. There is an opinion that this increase is good for liquidity by increasing providing liquidity of market maker strategies (MM) who earn profits from an order spread they made both sell and buy waiting orders (Angel et al. (2015); Otsuka (2014)). On the other hand, there is also the opposite opinion that this speed causes socially wasteful arms race for speed and these costs are passed to other investors as execution costs (Farmer and Skouras (2012); Budish et al. (2015)).

Budish et al. (2015) proposed a frequent batch auction (FBA) which reduces the value of speed advantages to terminate the socially wasteful arms race for speed. Many financial exchanges adopt a continuance double auction (CDA) in which multiple buyers and sellers compete to buy and sell some financial assets and where transactions can occur at any time whenever an offer to buy and an offer to sell match. On the other hand, on FBA buy and sell orders are grouped together and then executed every specific time intervals, for example some minutes, rather than executed one by one continuously. Budish et al. (2015) argued that these intervals leads to reduce the value of speed advantages and terminates the socially wasteful arms race for speed.

Budish et al. (2015) also analyzed using a simple model and empirical data, and argued that on CDA at high-frequency time horizons return correlations completely break down which leads to obvious mechanical arbitrage opportunities, on the other hand, on FBA the opportunities will be reduced. Fricke and Gerig (2015) discussed an optimal batch auction interval to minimize volatilities (standard deviations of returns) using a simple model. Manahov (2016) also discussed an optimal batch auction interval to prevent latency arbitrages calculating a time scale of latency arbitrages by empirical study using a simple model.

On the other hand, FBA is also criticized. Otsuka (2014) argued that on FBA market maker strategies (MM) providing liquidity are exposed to more risk because estimation of executed prices is more difficult, and then MM cannot continue to provide liquidity, therefore, liquidity will be reduced. This will leads to increase execution costs of other investors by bid ask spread becoming wider and will leads to reduce trading opportunities^{*1}.

Indeed, the models of Budish et al. (2015); Fricke and Gerig (2015) did not treat profits/loses of MM providing liquidity and so the models cannot discuss about possibility that MM cannot continue provide liquidity suggested by Otsuka (2014).

Bellia et al. (2015) studied empirically the possibility that MM cannot continue provide liquidity. As useful references but not exactly same as FBA, Bellia et al. (2015) compared data of batch

^{*1} Otsuka (2014) also argued that FBA will cause more serious arms race for speed because many orders will rushed within immediately before interval auctions.

auctions at opening and ending markets with that of CDA in the Tokyo stock exchange, and showed that MM made more orders on CDA than on batch auctions. This study implied that it is very important for MM that orders are immediately executed, FBA will lead to reduce traders who provides liquidity.

Empirical studies cannot be conducted to investigate situations that have never occurred in actual financial markets, changing from CDA to FBA. And ,so many factors cause price formation and liquidity in actual markets, an empirical study cannot be conducted to isolate the direct effect of latency to price formation.

Artificial market simulation^{*2} using a kind of agent based model can isolate the pure contribution of the changes to the price formation and can treat the changes that have never been employed. These are strong points of artificial market simulation studies.

Not only academies but also financial regulators and stock exchanges are recently interested in multi-agent simulations such artificial market models to investigate regulations and rules of financial markets. Indeed, the Science article, Battiston et al. (2016) described that ‘since the 2008 crisis, there has been increasing interest in using ideas from complexity theory (using network models, multi-agent models, and so on) to make sense of economic and financial markets’.

Recently, many artificial market studies contributed to discussion what financial regulations and rules should be. For example, price variation limits and short selling regulation whether preventing bubbles and crushes or not (Yagi et al. (2010); Yeh and Yang (2010); Mizuta et al. (2013, 2016a); Zhang et al. (2016)), transaction taxes (Westerhoff (2008)), financial leverages (Thurner et al. (2012); Veld (2016)), circuit breakers (Kobayashi and Hashimoto (2011)), usage rate of dark pools(Mo and Yang (2013); Mizuta et al. (2015)), cancel order tax (Veryzhenko et al. (2016)) and the effects of different regulatory policies directed towards high frequency traders (HFTs) (Leal and Napoletano (2016))^{*3}.

In the JPX Working Paper series, there are some studies contributed to discussion what financial regulations and rules should be. Mizuta et al. (2013) investigated effects of changing tick sizes, Kusada et al. (2014, 2015) investigated effects of market maker strategies and Mizuta et al. (2015, 2016b) investigated effects of increasing speed of order matching systems on financial exchanges.

However, it has not been investigated by artificial market simulations whether MM can continue to provide liquidity even on FBA.

^{*2} There are many excellent reviews(LeBaron (2006); Chen et al. (2012); Cristelli (2014); Mizuta (2016)). And we explain the basic concept for constructing our artificial market model in the Appendix “Basic Concept for Constructing Model”

^{*3} Of course, many artificial market simulation studies investigated the nature of financial markets, for examples, market impacts (Cui and Brabazon (2012); Oesch (2014)), financial market crash (Yagi et al. (2012); Paddrik et al. (2012); Mizuta et al. (2013); Torii et al. (2015); Schmitt and Westerhoff (2016)), interaction between option markets and underlying markets (Kawakubo et al. (2014a,b)), effects of passive funds (Braun-Munzinger et al. (2016)), and effects of HFTs (Gsell (2009); Wang et al. (2013); Xiong et al. (2015); Hanson and Hanson (2016)). Kita et al. (2016) reviewed the U-Mart project which is one of Japanese top artificial market research projects in the 2000s.

Therefore, in this study we implemented a price mechanism that is changeable between CDA ($\delta t = 1$) and FBA ($\delta t > 1$) continuously introducing a new parameter, a batch auction interval δt , based on the model of Kusada et al. (2014, 2015). And then, we analyzed profits/losses and risks of MM and investigated whether MM can continue to provide liquidity even on FBA by using an artificial market model.

2 Artificial Market Model

Chiarella and Iori (2002) is very simple but replicates long-term statistical characteristics observed in actual financial markets: fat-tail and volatility clustering. Mizuta et al. (2013) replicates high-frequency micro structures, such as execution rates, cancel rates, and one-tick volatility, which cannot be replicated with Chiarella and Iori (2002). Kusada et al. (2014, 2015) implemented market maker strategy agent (MM) based on the model of Mizuta et al. (2013).

In this study we implemented a price mechanism that is changeable between a comparable continuance double auction (CDA, $\delta t = 1$) and a frequent batch auction (FBA, $\delta t > 1$) continuously introducing a new parameter, a batch auction interval δt , based on the model of Kusada et al. (2014, 2015). The simplicity of the model is very important for this study because an unnecessary replication of macro phenomena leads to models that are over-fitted and too complex and such models would prevent our understanding and discovering mechanisms affecting the price formation because of related factors increasing. We explain the basic concept for constructing our artificial market model in the Appendix “Basic Concept for Constructing Model”.

2.1 Price Mechanism

In this study we implemented a price mechanism that is changeable between CDA ($\delta t = 1$) and FBA($\delta t > 1$) continuously. CDA is an auction mechanism in which multiple buyers and sellers compete to buy and sell some financial assets and where transactions can occur at any time whenever an offer to buy and an offer to sell match. On FBA, traders can make orders anytime but the orders are not executed for a batch auction interval δt . After the interval, buy orders in descending order of price and sell orders in ascending order of price are matched in turn and executed. We call this process a batch auction.

In our model t passes every normal agent (NA) ordered even if no deals have occurred. The batch auction interval is defined as δt , and we constructed the price mechanism model that is exactly same as CDA when $\delta t = 1$. When a batch auction is done at t , a market price (traded price) P^t is determined as the mid-price (the average price between the highest buy and lowest sell orders) among remained orders after the batch auction. And also when a batch auction is not done at t , P^t is determined as a tentative market price that the market price if a batch auction were done at t . This definition allows P^t is calculated continuously for any t .

	New Order -> time t=0	Sell 99 t=1	Buy 100 t=2	Buy 101 t=3	Sell 98 t=4
CDA $\delta t=1$	<u>Sell Price Buy</u>	<u>Sell Price Buy</u>	<u>Sell Price Buy</u>	<u>Sell Price Buy</u>	<u>Sell Price Buy</u>
	1 101	1 101	1 101	1 101 1	1 101
	1 100	1 100	1 100 1	1 100 1	1 100
	99 1	1 99 1	99 1	99 1	99 1
	98 1	98 1	98 1	98 1	1 98 1
		Immediately Executed	Immediately Executed	Immediately Executed	Immediately Executed
FBA $\delta t=4$	<u>Sell Price Buy</u>	<u>Sell Price Buy</u>	<u>Sell Price Buy</u>	<u>Sell Price Buy</u>	<u>Sell Price Buy</u>
	1 101	1 101	1 101	1 101 1	1 101 1
	1 100	1 100	1 100 1	1 100 1	1 100 1
	99 1	1 99 1	1 99 1	1 99 1	1 99 1
	98 1	98 1	98 1	98 1	1 98 1
		Not Executed	Not Executed	Not Executed	Executed at specific time

Figure 1 Example for price mechanism

Figure 1 shows an example for price mechanism. The top shows the case of $\delta t = 1$ (CDA) and the bottom shows the case of $\delta t = 4$ (FBA). From the left side to right at $t = 0, 1, 2, 3, 4$ order books are showed. $t = 0$ is immediately after a batch auction and remained orders are same for both cases. Here, at $t = 1$ a new sell order at a price 99 is made. In the case of $\delta t = 1$, the new sell order is matched to the same price as buy order immediately and both orders are banished. On the other hand, the case of $\delta t = 4$, the new sell order remains because of $t = 1$ is not a batch auction time. In the same way, new buy orders at prices 100 and 101 are made at $t = 2, 3$, respectively, in the case of $\delta t = 1$ both orders are executed, on the other hand, in the case of $\delta t = 4$ both orders remains. At $t = 4$, a new sell order at the price 98 is made, in the case of $\delta t = 1$ the order is executed as the same way. In the case of $\delta t = 4$, remember $t = 4$ is a batch auction time, buy orders in descending order of price and sell orders in ascending order of price are matched in turn. As a result, buy order of a price 101 and sell 100, and buy 100 and sell 99 are matched, respectively. A market price P^t is determined as the mid-price 99.5.

Note that remaining orders are very different between the case of $\delta t = 1$ and that of $\delta t = 4$. In the case of $\delta t = 1$, no orders remain, on the other hand, in the case of $\delta t = 4$, four orders remain. Furthermore, in the case of $\delta t = 1$, 4 buy orders are executed in all, on the other hand in the case of $\delta t = 4$, only 2 buy orders are executed in all. In this way, price formations and order books depend on δt .

2.2 Agent

Our model treats only one risk asset. The numbers of normal agents (NA) and market maker agents (MM) are n and 1, respectively. First, at time $t = 1$, NA no. 1 orders to buy or sell the risk asset; then at $t = 2, 3, \dots, n$, NA no. 2, 3, ..., n respectively order to buy or sell. MM makes two orders, buy and sell immediately before NA ordering. As shown in Figure 2, MM orders every time step by next batch auction and immediately after the batch auction all MM's orders are canceled. At $t = n + 1$, going back to the first NA, NA 1 orders to buy or sell, and at $t = n + 2, n + 3, \dots, n + n$, NA no. 2, 3, ..., n respectively order to buy or sell, and this cycle is repeated. Note that t passes every NA ordered even if no deals have occurred.

Agents always order only one share. The quantity of holding positions is not limited, so agents can take any shares for long and short positions to infinity. The minimum unit of price change is δP . The buy order price is rounded off to the nearest fraction, and the sell order price is rounded up to the nearest fraction.

2.2.1 Normal Agent (NA)

To replicate nature of price formations in actual financial markets we introduced a normal agent (NA), and we modeled NA as very general investors and as simple as replicating long-term statistical characteristics and very short term micro structure in real financial markets.

NA determines an order price and buys or sells as follows. NA uses a combination of a fundamental value and technical rules to form expectations on a risk asset's returns. An expected return of agent j is

$$r_{e,j}^t = \frac{1}{w_{1,j} + w_{2,j} + u_j} \left(w_{1,j} \log \frac{P_f}{P^t} + w_{2,j} r_{h,j}^t + u_j \epsilon_j^t \right), \quad (1)$$

where $w_{i,j}$ is the weight of term i of agent j and is independently determined by random variables uniformly distributed in the interval $(0, w_{i,max})$ at the start of the simulation for each agent, u_j is the weight of the third term of agent j and is also independently determined by random variables uniformly distributed in $(0, u_{max})$ at the start of the simulation for each agent, P_f is a fundamental value that is constant^{*4}, P^t is a market price (a tentative market price) defined on the previous section, ϵ_j^t is noise determined by random variables of normal distribution with an average 0 and a variance σ_ϵ , $r_{h,j}^t$ is a historical price return inside an agent's time interval τ_j , and $r_{h,j}^t = \log(P^t / P^{t-\tau_j})$, and τ_j is independently determined by random variables uniformly distributed in the interval $(1, \tau_{max})$ at the start of the simulation for each agent^{*5}.

The first term of Eq. (1) represents a fundamental strategy: an agent expects a positive return when the market price is lower than the fundamental value, and vice versa. The second term of

^{*4} We focused phenomena in time scale as short as the fundamental price remains static.

^{*5} However, when $t < \tau_j$, $r_{h,j}^t = 0$.

Order both Sell and Buy at once	Order every time by a batch auction																																																																														
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Figure 2 Orders of the market maker agent (MM)

Eq. (1) represents a technical strategy: an agent expects a positive return when historical market return is positive, and vice versa.

After the expected return has been determined, the expected price is

$$P_{e,j}^t = P^t \exp(r_{e,j}^t). \quad (2)$$

An order price $P_{o,j}^t$ is determined by random variables normally distributed in an average $P_{e,j}^t$ and a standard deviation P_σ , where P_σ is a constant.

Buy or sell is determined by the magnitude relationship between $P_{e,j}^t$ and $P_{o,j}^t$, i.e.,

when $P_{e,j}^t > P_{o,j}^t$, the agent orders to buy one share,

when $P_{e,j}^t < P_{o,j}^t$, the agent orders to sell one share^{*6}.

2.2.2 Market Maker Agent (MM)

In this study, we constructed a model of a Market Maker Agent (MM) based on the models of Kusada et al. (2014, 2015). As Figure 2 shows, MM makes one sell order in the price higher by an order spread P_{spread} than a fair value P_{fair} , in short $P_{fair} + P_{spread}$, and makes one buy order in the price $P_{fair} - P_{spread}$. MM makes these orders immediately before NA ordering, and orders every time step by next batch auction. This leads that amount of orders of MM is constant and is independent on δt . Immediately after the batch auction all MM's orders are canceled.

We implemented four kinds of MM. First one is a simple MM (SMM) with $P_{fair} = P^t$. Second one is a position MM (PMM) with

$$P_{fair} = (1 - kS^3)P^t, \quad (3)$$

^{*6} However, during $t < t_c$, due to make enough waiting orders, when $P_f > P_{o,j}^t$, the agent orders to buy one share, when $P_f < P_{o,j}^t$, the agent orders to sell one share.

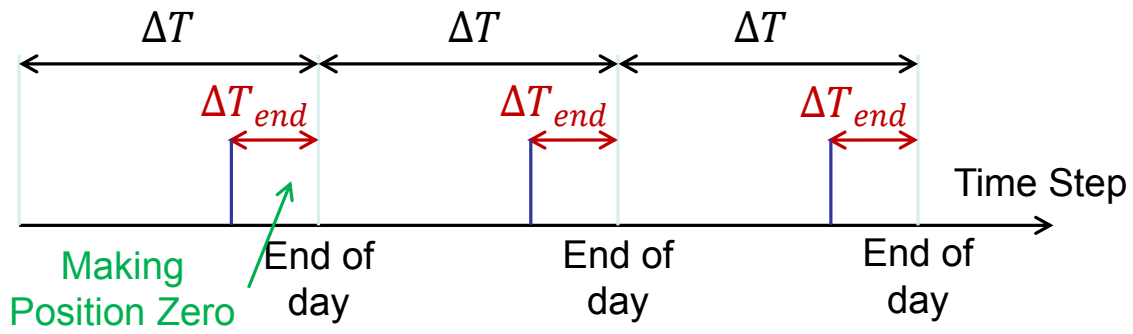


Figure 3 Periods that MM is closing its position

<p><u>PMM3</u> <u>Do not order increasing position</u> In the case of negative position, within last 2,000 time steps</p> <table border="0"> <thead> <tr> <th style="text-align: left;">Sell</th> <th style="text-align: left;">Price</th> <th style="text-align: left;">Buy</th> </tr> </thead> <tbody> <tr> <td></td> <td>10011</td> <td></td> </tr> <tr> <td>Do not order</td> <td>10010</td> <td></td> </tr> <tr> <td></td> <td>10009</td> <td>← P_{fair}</td> </tr> <tr> <td></td> <td>10008</td> <td></td> </tr> <tr> <td></td> <td>10007</td> <td>1</td> </tr> </tbody> </table>	Sell	Price	Buy		10011		Do not order	10010			10009	← P _{fair}		10008			10007	1	<p><u>PMM4</u> <u>Change order price that of opposite side (buy/sell)</u> In the case of negative position, within last 2,000 time steps</p> <table border="0"> <thead> <tr> <th style="text-align: left;">Sell</th> <th style="text-align: left;">Price</th> <th style="text-align: left;">Buy</th> </tr> </thead> <tbody> <tr> <td></td> <td>10011</td> <td>1 ← change order price here</td> </tr> <tr> <td></td> <td>10010</td> <td></td> </tr> <tr> <td></td> <td>10009</td> <td>← P_{fair}</td> </tr> <tr> <td></td> <td>10008</td> <td></td> </tr> <tr> <td></td> <td>10007</td> <td></td> </tr> </tbody> </table>	Sell	Price	Buy		10011	1 ← change order price here		10010			10009	← P _{fair}		10008			10007	
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Figure 4 Orders of PMM3 and PMM4 on the position closing period

where k is a constant, S is a holding position of MM. In the case of $S > 0$, P_{fair} is smaller than P^t , then order prices for both buy and sell are lower. Therefore, it will be more difficult for buy orders to be executed and easier for sell orders, and then the absolute of S ($\|S\|$) has tendency to decrease. On the other hand in the case of $S < 0$, P_{fair} is higher than P^t , then order prices for both buy and sell are higher. Therefore, it will be easier for buy orders to be executed and more difficult for sell orders, and then $\|S\|$ has tendency to decrease. In both case, $\|S\|$ of PMM is smaller than that of SMM, so PMM reduce profit/loss risks.

Actual MMs also make effort to reduce their position to avoid profit/loss risks. Especially, they usually close all position at the end of a day because if there are big head lines on overnight (from the end of a day to the start of next day), they could lead to take large loss. Therefore, we added more two kinds of MMs that try to avoid such the overnight risk unconsidered in Kusada et al. (2014, 2015).

Figure 3 shows periods that MM is closing its position and a definition of the length of one day. We defined the length of one day as ΔT , and constructed two more kind of MM, PMM3 and PMM4 which make effort to reduce their positions on the last period ΔT_{end} in one day (position closing period) to hold no overnight position. The left side on Figure 4 shows orders of PMM3 on the position closing periods. On the periods, PMM3 make no buy order when $S > 0$ and no

sell order when $S < 0$. The right side on Figure 4 shows orders of PMM4 on the position closing periods. On the periods, PMM4 make a buy order in the price $P_{fair} - P_{spread}$ that is normally sell order price when $S > 0$ and a sell order in the price $P_{fair} + P_{spread}$ that is normally buy order price when $S < 0$.

PMM4 makes orders in the more aggressive prices than PMM3 to reduce its position. Such orders of PMM4 match waiting orders of NA, and this means taking liquidity providing by NA. And then, when $\|S\|$ is large, PMM4 disturbs market prices to one direction and a price variation risk of PMM4 is higher. Therefore, in actual market, PMM3 is more actual than PMM4, however, when δt is large PMM3 cannot make its overnight position zero as showed on the next section. For this reason, we constructed PMM4.

3 Simulation Result

In this study, we set the same parameters as those of Mizuta et al. (2013) and Kusada et al. (2014, 2015). Specifically, we set^{*7} $n = 1,000, w_{1,max} = 1, w_{2,max} = 10, u_{max} = 1, \tau_{max} = 10,000, \sigma_\epsilon = 0.06, P_\sigma = 30, t_c = 20,000, \delta P = 0.02, P_f = 10,000, k = 0.00000005, \Delta T = 20,000, \Delta T_{end} = 2,000$. And we ran simulations at $t = t_e = 10,000,000$.

And we ran simulations for four kinds of MM, $P_{spread}/P_f = 0.03\%, 0.1\%, 0.3\%, 1\%$, and $\delta t = 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000$ ^{*8}, not only under other parameters that were fixed but also the same random number table. We simulated these runs 100 times, changing the random number table each time, and used averaged statistical values of 100 runs.

3.1 Order Spread (P_{spread}/P_f) and Execution Ratio

Table 1 shows execution rates of MM (PMM4) for various batch auction intervals (δt) and order spreads of MM (P_{spread}/P_f). An execution rate is defined as number of executed orders per number of all orders.

δt is larger, execution rates of MM is smaller. These decreasing execution rates cause to reduce liquidity supply by MM, because MM provides liquidity (opportunities of trades) to NA always making both sell and buy waiting orders. Therefore, δt is smaller, liquidity is more provided. This implies that MM can provide liquidity most on CDA.

P_{spread}/P_f is larger, of course, execution rates of MM is smaller. When $P_{spread}/P_f = 0.1\%$, at $\delta t = 1$ the execution rates are realistic, however, at $\delta t \geq 50$ the execution rates are little and it is impossible to analyze holding position of MM, profit/loss risks and so on. Therefore, we used $P_{spread}/P_f = 0.03\%$ following sections to enable meaningful analysis even though $\delta t \geq 50$.

^{*7} We explain how they verified their model in the Appendix "Verification of the Model".

^{*8} In short, we simulated 160(= $4 \times 4 \times 10$) cases by 4 kinds of MM, 4 cases of P_{spread} and 10 cases δt .

Table 1 Execution rates of MM (PMM4) for various batch auction intervals (δt) and order spreads of MM (P_{spread}/P_f)

Execution Rate of MM		P_{spread}/P_f			
		0.03%	0.10%	0.30%	1.00%
δt	1(CDA)	8.06%	1.53%	0.00%	0.00%
	2	6.30%	0.88%	0.00%	0.00%
	5	3.93%	0.37%	0.00%	0.00%
	10	2.47%	0.14%	0.00%	0.00%
	20	1.49%	0.02%	0.00%	0.00%
	50	0.77%	0.00%	0.00%	0.00%
	100	0.48%	0.00%	0.00%	0.00%
	200	0.32%	0.00%	0.00%	0.00%
	500	0.21%	0.00%	0.00%	0.00%
	1000	0.22%	0.00%	0.00%	0.00%

Table 2 Averages of $\|S\|$ within whole period or end periods on a day for various batch auction intervals (δt) and kinds of MM. ($P_{spread}/P_f = 0.03\%$)

Average of $\ S\ $	SMM		PMM		PMM3		PMM4		
	Whole Period	End	Whole Period	End	Whole Period	End	Whole Period	End	
		Period on a day		Period on a day		Period on a day		Period on a day	
δt	1(CDA)	12,357	12,371	3.18	3.08	2.90	0.00	2.89	0.00
	2	17,42	17,441	3.10	3.25	2.79	0.00	2.79	0.00
	5	4,409	4,414	3.87	3.95	3.48	0.00	3.48	0.00
	10	1,744	1,744	4.44	4.34	4.01	0.02	3.96	0.00
	20	548	548	4.84	4.71	4.52	0.78	4.35	0.00
	50	384	385	5.27	5.14	5.02	2.63	4.63	0.00
	100	369	370	5.57	5.51	5.56	4.26	4.80	0.00
	200	174	174	5.91	6.11	5.92	5.69	4.38	0.00
	500	72	71	5.75	6.06	5.70	5.81	2.32	0.03
	1000	290	290	5.94	6.11	5.61	5.80	1.76	0.06

3.2 Position for various kinds of MM

Table 2 shows averages of $\|S\|$ within whole period or end periods on a day for various δt and kinds of MM at $P_{spread}/P_f = 0.03\%$.

Table 3 Final profits of MM, averages of $\|S\|$ and execution rates of MM and NA for various batch auction intervals (δt). (PMM4, $P_{spread}/P_f = 0.03\%$)

	Final Profit of MM / P_f	Average of $\ S\ $			
		Whole Period	End Period on a day	Execution Rate of MM	Execution Rate of NA
1(CDA)	51.98	2.89	0.00	8.06%	39.1%
2	-29.42	2.79	0.00	6.30%	39.1%
5	-14.90	3.48	0.00	3.93%	37.6%
10	-4.08	3.96	0.00	2.47%	36.3%
δt 20	1.51	4.35	0.00	1.49%	34.9%
50	3.68	4.63	0.00	0.77%	33.4%
100	2.53	4.80	0.00	0.48%	32.5%
200	0.93	4.38	0.00	0.32%	31.8%
500	-0.06	2.32	0.03	0.21%	31.0%
1000	-0.10	1.76	0.06	0.22%	30.5%

Actual MMs also make effort to reduce their position to avoid profit/loss risks. Especially, they usually close all position at the end of a day because if there are big head lines on overnight (from the end of a day to the start of next day), they could lead to take large loss.

Therefore, it is unrealistic that the case of large averages of $\|S\|$ within end periods on a day because of high loss risks of MM. SMM and PMM which are not implemented to make effort to close all position at the end of a day, of course, have unrealistic large $\|S\|$.

When $\delta t \leq 10$, $\|S\|$ of PMM3 within end periods on a day are almost zero, and so PMM3 can prevent overnight risks. On the other hand, when $\delta t \geq 20$ PMM3 cannot make $\|S\| = 0$, and PMM3 is exposed unrealistic high risk on overnights. Furthermore, as we mentioned previous section, δt is larger, execution rates are smaller, none the less $\|S\|$ within whole period is larger and PMM3 takes more loss risks. In short, when $\delta t \geq 20$ PMM3 cannot avoid both an overnight risk and a price variation risk intraday, this suggests that PMM3 cannot continue to provide liquidity.

Only PMM4 which makes orders in the more aggressive prices to reduce its position within end periods on a day can make its position almost zero even when $\delta t \geq 20$. Such orders of PMM4 match waiting orders of NA, and this means taking liquidity providing by NA. And then, when $\|S\|$ is large, PMM4 disturbs market prices to one direction and a price variation risk of PMM4 is higher. However, otherwise MM cannot prevent overnight risks when $\delta t \geq 20$. This implies that when δt is large (FBA), MM should take another risks to avoid overnight risks and cannot continue to provide liquidity. However, we used PMM4 following section because only PMM4

avoid overnight risks.

3.3 Final Profit

Table 3 shows final ($t = t_e$) profits of MM (PMM4), averages of $\|S\|$ and execution rates of MM and NA for various δt at $P_{spread}/P_f = 0.03\%$.

When $\delta t = 2, 5, 10$, profits of MM are negative and MM lost money. In this study, we treat a very steady market in that MM earns profit easily. None the less losing money means it is very difficult for MM to earn profits in actual market. When $\delta t = 20, 50, 100, 200$, MM cannot also earn enough, the profits are much smaller than that when $\delta t = 1$ (CDA). As we mentioned previously δt is larger, price variation risks of MM are higher. Considering these factors, when $\delta t > 1$ (FBA) it is very difficult that MM is rewarded for risks and continue to provide liquidity.

4 Summary and Future Works

In this study we implemented a price mechanism that is changeable between a comparable continuance double auction (CDA, $\delta t = 1$) and a frequent batch auction (FBA, $\delta t > 1$) continuously introducing a new parameter, a batch auction interval δt , based on the model of Kusada et al. (2014, 2015). And then, we analyzed profits/losses and risks of market maker strategies (MM) and investigated whether MM can continue to provide liquidity even on FBA by using an artificial market model.

Our simulation results showed that δt is larger, execution rates of MM is smaller and this causes to reduce liquidity supply by MM. Furthermore, they suggested that when δt is larger (FBA), MM cannot avoid both an overnight risk and a price variation risk intraday. Furthermore, they also suggested that when $\delta t > 1$ (FBA) it is very difficult that MM is rewarded for risks and continue to provide liquidity. Only the case of $\delta t = 1$ (CDA) MM is rewarded for risks and continue to provide liquidity.

These suggestions implies that MM that can provide liquidity on CDA cannot continue to provide liquidity on FBA and then many MM retire, and finally liquidity will be reduced. This implication is consistent with the argument by Otsuka (2014).

One of future work is a discussion for more suitable MM to FBA. In this study, we showed the possibility MM adapting to CDA is not suitable to FBA. An existence of MM adapting to FBA is unlikely, however, this study cannot outright deny the existence. Another future work is an investigation the case of a few waiting orders or of no MM providing liquidity. In the case of a few waiting orders on the order book, it is possible that investors can make order easier on FBA than on CDA. The case of no MM is off table of this study.

For more detailed discussions, we should compare the simulation results to those from studies using other methods, e.g., empirical studies and theoretical studies. An artificial market can

isolate the direct effect of changes in market systems to price formation, and can treat situations that have never occurred. However, outputs of artificial market simulations may not be accurate or credible forecasts in actual markets. It is an important for artificial market simulations to show possible mechanisms affecting price formation through many runs and gain new insight; conversely, a limitation of artificial market simulations is that their outputs may, but not certainly, occur in actual financial markets.

Appendix

4.1 Basic Concept for Constructing Model

An artificial market, which is a kind of agent based models, can isolate the pure contribution of these system changes to the price formation and can treat the changes that have never been employed (LeBaron (2006); Chen et al. (2012); Cristelli (2014); Mizuta (2016)). These are the strong points of the artificial market simulation study.

However, outputs of the artificial market simulation study would not be accurate or credible forecasts of the actual future. The artificial market simulation needs to show possible mechanisms affecting the price formation by many simulation runs, e.g. searching for parameters, purely comparing before/after the changing, and so on. The possible mechanisms shown by these simulation runs will give us new intelligence and insight about effects of the changes to price formation in actual financial markets. Other study methods, e.g. empirical studies, would not show such possible mechanisms.

Indeed, artificial markets should replicate macro phenomena existing generally for any asset and any time. Price variation, which is a kind of macro phenomena, is not explicitly modeled in artificial markets. Only micro processes, agents (general investors), and price determination mechanisms (financial exchanges) are explicitly modeled in artificial markets. Macro phenomena are emerging as the outcome interactions of micro processes. Therefore, the simulation outputs should replicate general macro phenomena at least to show that simulation models are probable in actual markets.

However, it is not a primary purpose for the artificial market to replicate specific macro phenomena only for a specific asset or a specific period. An unnecessary replication of macro phenomena leads to models that are over-fitted and too complex. Such models would prevent our understanding and discovering mechanisms affecting the price formation because of related factors increasing.

Indeed, artificial market models that are too complex are often criticized because they are very difficult to evaluate (Chen et al. (2012)). A too complex model not only would prevent our understanding mechanisms but also could output arbitrary results by over-fitting too many parameters. Simpler models harder obtain arbitrary results, and are easier evaluated.

Table 4 Statistics without MM on $\delta t = 1(\text{CDA})$

	execution rate	32.3%
trading	cancel rate	26.1%
	number of trades / 1 day	6467
standard	for 1 tick	0.0512%
deviations	for 1 day (20000 ticks)	0.562%
	kurtosis	1.42
	lag	
	1	0.225
autocorrelation	2	0.138
coefficient for	3	0.106
square return	4	0.087
	5	0.075

Therefore, we constructed an artificial market model that is as simple as possible and do not intentionally implement agents to cover all the investors who would exist in actual financial markets.

4.2 Verification of the Model

In many previous artificial market studies, the models were verified to see whether they could explain stylized facts such as a fat-tail, volatility-clustering, and so on (LeBaron (2006); Chen et al. (2012); Cristelli (2014); Mizuta (2016)). A fat-tail means that the kurtosis of price returns is positive. Volatility-clustering means that the square returns have positive autocorrelation, and the autocorrelation slowly decays as its lag becomes longer. Many empirical studies, e.g. that of Sewell (2011), have shown that both stylized facts (the fat-tail and volatility-clustering) exist statistically in almost all financial markets. Conversely, they also have shown that only the fat-tail and volatility-clustering are stably observed for any asset and in any period because financial markets are generally unstable.

Indeed, the kurtosis of price returns and the autocorrelation of the square returns are stably and significantly positive, but the magnitudes of these values are unstable and very different depending on asset and/or period. The kurtosis of price returns and the autocorrelation of the square returns were observed to have very broad magnitudes of about $1 \sim 100$ and about $0.01 \sim 0.2$, respectively (Sewell (2011)).

For the above reasons, an artificial market model should replicate these values as significantly positive and within a reasonable range as we mentioned. It is not essential for the models to replicate specific values of stylized facts because these stylized facts' values are unstable in actual

financial markets.

Table 4 lists statistics in which there is no market maker strategy on $\delta t = 1$ (a continuance double auction). All statistics are averages of 100 simulation runs, and all the following figures use the average of 100 simulation runs. We define 20,000 time steps as 1 day because the number of trades within 20,000 time steps is almost the same as that in actual markets per day. All statistics; execution rates, cancel rates^{*9}, standard deviations of returns for one tick and one day^{*10}, kurtosis of price returns, and the autocorrelation coefficient for square returns^{*11} are of course almost the same as the results of actual financial markets. These showed that this mode replicated very short term micro structure, execution rates, cancel rates and standard deviations of returns for one tick, and replicated long-term statistical characteristics, fat tail, and volatility clustering, observed in real financial markets. Therefore, the model was verified to investigate whether market maker strategies can continue to provide liquidity even on a frequent batch auction.

^{*9} The execution rate is the ratio of the number of trades to that of all orders. The cancel rate is the ratio of the number of cancels to that of all orders + cancels.

^{*10} In our model, though overnight returns do not exist, the standard deviations of returns for one day correspond to intraday volatility in real financial markets.

^{*11} We used returns for 10 time units' intervals (corresponding to about 10 seconds) to calculate the statistical values for the stylized facts. In this model, time passes by an agent just ordering even if no dealing is done. Therefore, the returns for one tick (one time) include many zero returns, and they will bias statistical values. This is the reason we use returns for about 10 time units' intervals.

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