Price Variation Limits and Financial Market Bubbles: Artificial Market Simulations with Agents' Learning Process

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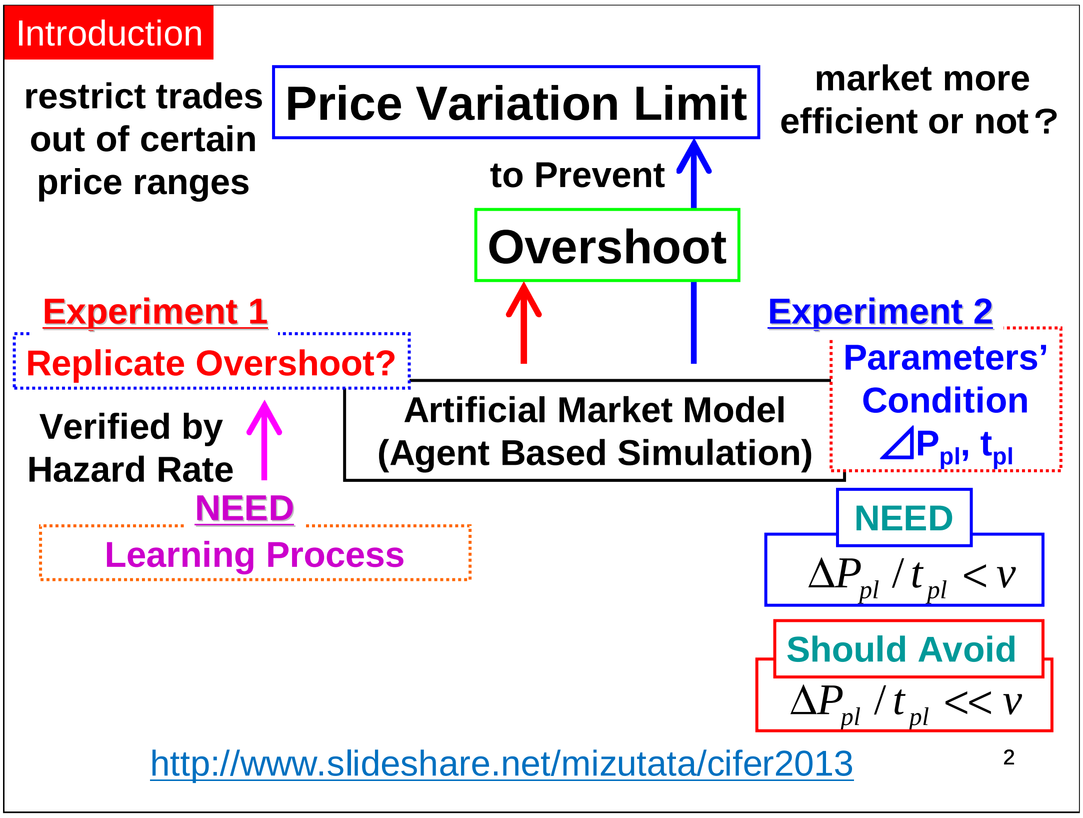
Thank you very much. I'm Takanobu MIZUTA from SPARX Asset Management.
I'm also belonging to The University of Tokyo.
Today, I'm going to give a presentation under the title of This.



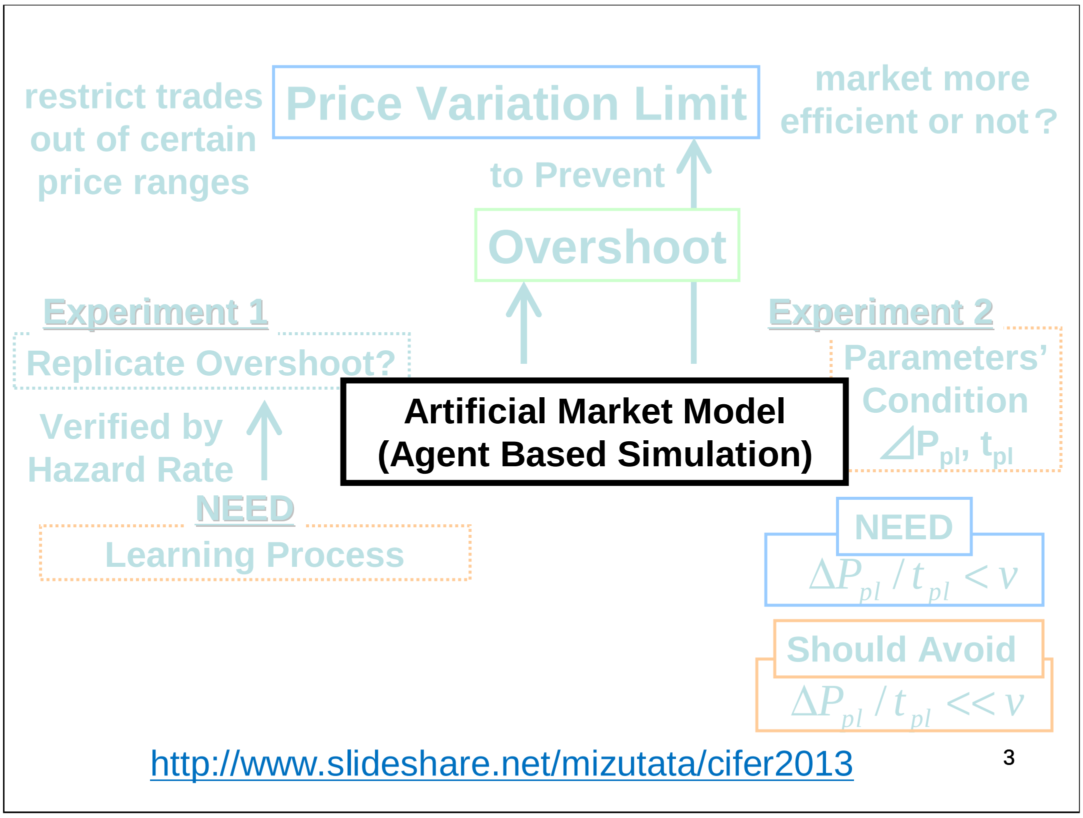
■ Financial exchanges sometimes employ a “price variation limit”, which restrict trades out of certain price ranges to avoid sudden large price fluctuations, “overshoots” overshoots occur in bubble and crash in real financial market

■ There is a debate over whether the price variation limit makes financial market more efficient or not.

■ To investigate price variation limits, we built an artificial market model implementing a learning process.

■ Experiment 1, we will show that the model should be implemented the learning process to replicate overshoot.

■ Experiment 2, We will also show that a parameters' condition of the price variation limit to prevent overshoot.



First, I will describe our artificial market model.

Artificial Market Model (Agent Based Model)

Chiarella et. al. [2009]

- **Continuous Double Auction**
  ⇒ to implement realistic price variation limit
- **Agent model is Simple**
  ⇒ to avoid arbitrary result “Keep it short and simple”

heterogeneous 1000 agents

Expected Return

$$r_{e,j}^{t} = \frac{1}{\sum_i w_{i,j}} \left(w_{1,j} \log \frac{P_f}{P^t} + w_{2,j} r_{h,j}^{t} + w_{3,j} \varepsilon_j^t \right)$$

Fundamental | Technical | noise

$w_{i,j}$ Strategy Weight ↑ Different for each agent

+ Learning Process **Our Original**

↑ need to replicate an overshoot

Good Performance Strategy $w_{i,j}$ is up

Bad Performance Strategy $w_{i,j}$ is down

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We built an artificial market model on basis of Chiarella et. al. [2009].

●Pricing mechanism is Continuous Double Auction

It is not simple, but, we need to implement realistic price variation limit

●Agent Model is Simple. This is to avoid arbitrary result, “Keep it short and simple” principle.

We think Artificial Market Models should explain Stylized Facts as Simply as possible.

There are heterogeneous 1000 agents. All agents calculate Expected Return using this equation,

And, the strategy weights are different for each agent

・First term is a Fundamental Strategy: When the market price is smaller than the fundamental price, an agent expects a positive return , and vice verse.

・Second term is a technical strategy: When historical return is positive, an agent expects a positive return, and vice verse.

・Third term is noise.

Chiarella’s model did not include Learning Process, however,

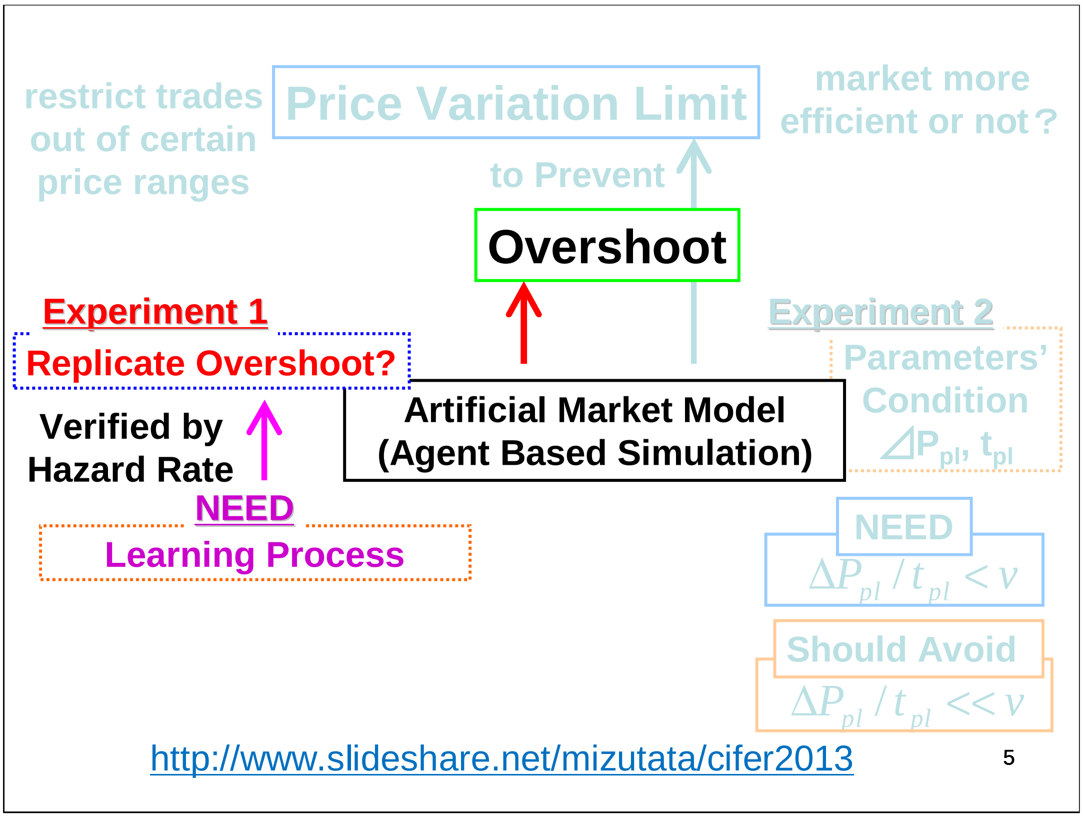
We built Learning Process of agents, this is our Original.

We showed that learning process need to replicate an overshoot.

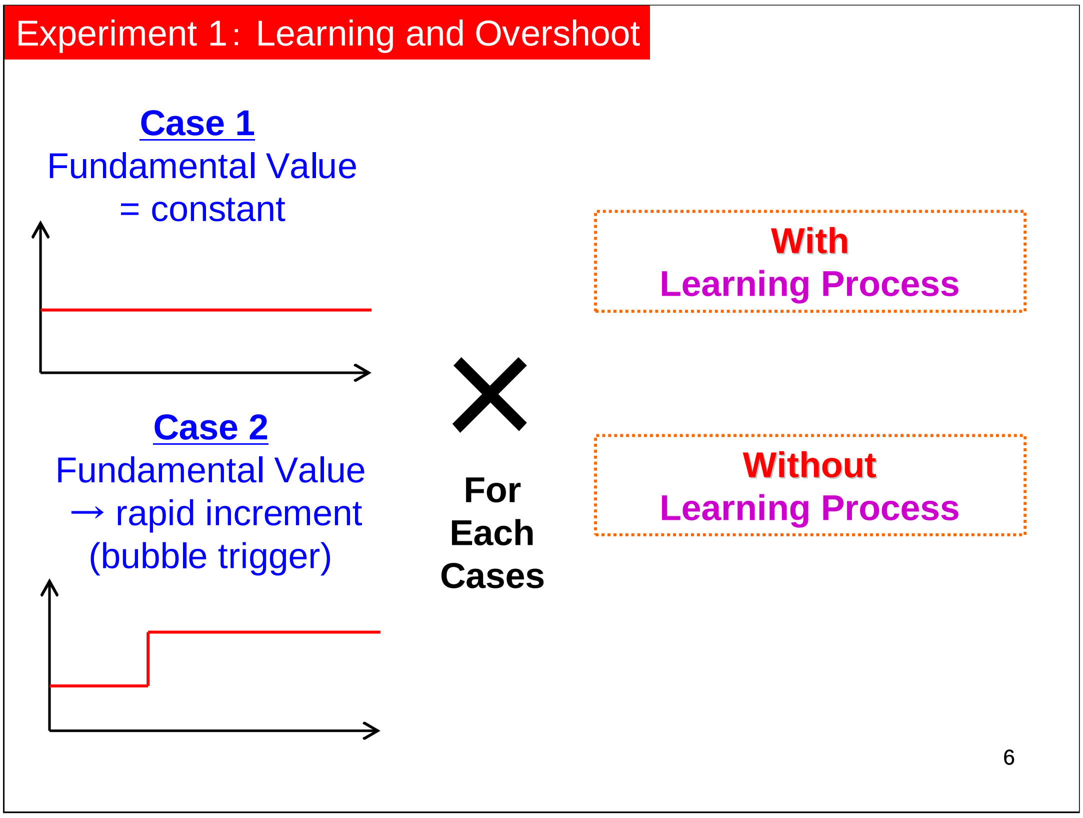
Agents are comparing Historical Return and Each Strategy’s Return.

・When the strategy’s return and Historical Return are Same Sign, Good Performance Strategy, The strategy’s Weight is Up.

・When the strategy’s return and Historical Return are Opposite Sign, Bad Performance Strategy, The strategy’s Weight is Down.



Next, I show simulation results of Experiment 1 about leaning process and replicating overshoot



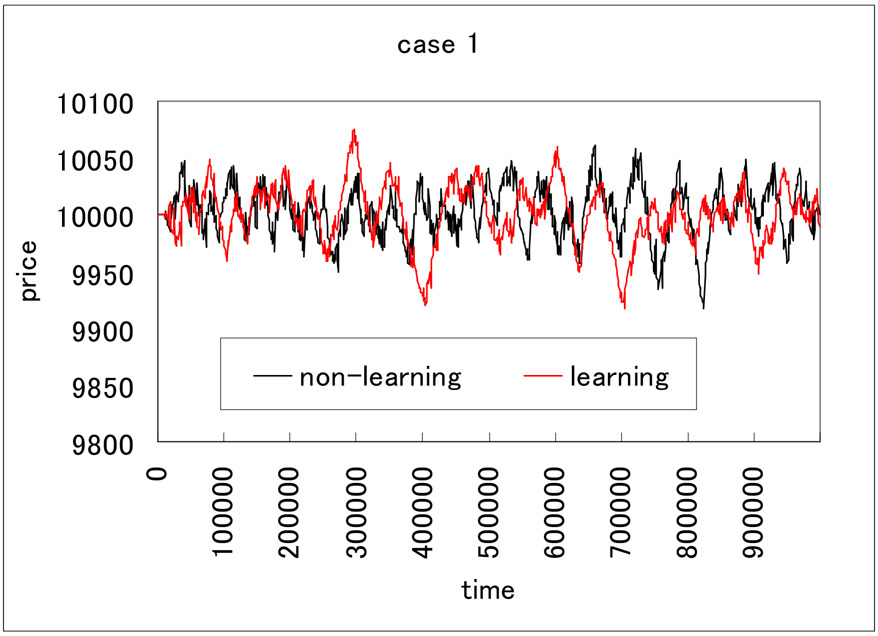
We examined two Cases, Case 1, Fundamental Value is constant,

Case 2, Fundamental value is rapid incremented like this. This is bubble inducing trigger.

For Each cases, we examined With learning process And WithOut learning process.

Therefore, we examined four cases in all.

Case 1: Fundamental Value = constant



Small fluctuating around Fundamental Value

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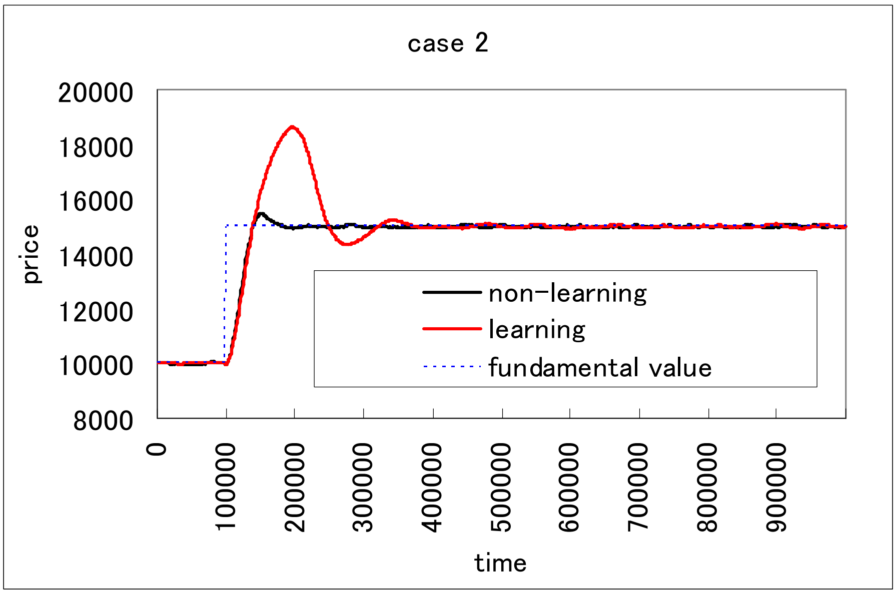
This Figure shows time evolution of market prices in case 1, Fundamental Value is constant.

In both cases, With learning process and without learning process. the results are very similar,

The prices were small fluctuating around Fundamental Value, here, Ten Thousand

Case 2 : Fundamental Value → rapid increment (bubble trigger)



Only with learning process, Overshoot occurred

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This Figure shows time evolution of prices in case 2.

Fundamental value was changed at this time, increased to New Fundamental Value, Fifteen Thousand.

This is the bubble inducing trigger.

Without Learning Process, Black line, Overshooting was not occurred.

On the other hand, with Learning Process, Red line,

the price went far beyond the new fundamental value.

Only with learning process, Overshoot occurred.

Traditional Stylized Facts

		case 1		case 2	
		non-learning	learning	non-learning	learning
kurtosis		3.018	5.394	2.079	3.180
	lag				
autocorrelation coefficient for square return	1	0.134	0.125	0.219	0.325
	2	0.101	0.105	0.164	0.293
	3	0.076	0.087	0.133	0.274
	4	0.060	0.074	0.118	0.261
	5	0.052	0.061	0.108	0.253
	6	0.040	0.054	0.100	0.247
	7	0.036	0.048	0.092	0.241
	8	0.030	0.045	0.087	0.237
	9	0.026	0.039	0.082	0.238

All cases replicated: Fat Tail and Volatility Clustering

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This Table lists Traditional stylized facts in each case.

In all cases, both kurtosis and autocorrelation for square returns for all i are positive.

This means that all cases replicate Traditional stylized facts: fat-tail and volatility-clustering.

Hazard Rate (similar to "run test")

New Stylized fact to verify model replicating overshoot

H(i) conditional probability that
sequence of positive return ends at *i*, given that it lasts until *i*.

For Example *i*=3、*H(3)*

Time ・・・・ 1 2 3 4

Return ↘ ↗ ↗ ↗ ↘

H(3): Probability of 4th Return Negative

Empirical Studies:
Any cases: *H(i)* < 50%,
Overshoot period: *H(i)* decline with *I* rapidly
McQueen and Thorley [1994], Chan et. al. [1998]
⇒ Overshoot returns tend to continue to be positive
this tendency stronger continuing positive returns longer

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We propose Hazard Rate as New Stylized fact to verify model replicating overshoot

Hazard Rate Hi is conditional probability that sequence of positive return ends at i, given that it lasts until i.

For Example i=3, H3 means like this.

1^{st}, positive return, 2^{nd}, positive, 3^{rd} positive,

In this condition, H3 is probability of 4^{th} return become negative.

Empirical Studies showed that, Any cases, Hi for most of i are smaller than 50%

And when including overshoot period, Hi decline rapidly with i,

This show that the overshoot returns tend to continue to be positive

And this tendency stronger continuing positive returns longer

New Stylized Facts: Hazard Rate H(i)

		case 1		case 2	
		non-learning	learning	non-learning	learning
	i				
hazard rate	1	56%	55%	56%	55%
	2	55%	52%	55%	50%
	3	55%	50%	53%	45%
	4	54%	49%	52%	40%
	5	54%	45%	48%	36%
	6	53%	44%	45%	29%
	7	52%	41%	40%	26%
	8	52%	40%	35%	22%
	9	53%	40%	30%	19%

Only with Learning process → Verified by Hazard Rate
And Only Case 2 with Learning ⇒ Replicating Overshoot

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This Table lists New Stylized Facts: Hazard Rate in each case.

In case 2 with learning, hazard rate declined rapidly.
This case can replicate a significant Overshoot like actual markets.
On the other hand, the case without learning, hazard rate dose not declined rapidly.
The case can not replicate Overshoot.

Case 1, without learning, Hazard rates are upper 50% for all i.
This is Not consistent with empirical study.
On the other hand, Case 1, with learning, Hazard rates for most of i are smaller than 50%, even when price fluctuations are stable.

This consistent with empirical study.

Therefore, only cases with Learning Process were verified by Hazard Rate,
and only Case 2 can replicate overshoot.

Result Summary Experiment 1： Learning and Overshoot

	Not-Consistent with Empirical study ↑ Without Learning Process	Consistent with Empirical study ↑ With Learning Process
Case1 Fundamental Value = constant	Stable Not-Verified by Hazard Rate	Stable Verified by Hazard Rate
Case2 Fundamental Value → rapid increment	No-Overshoot Not-Verified by Hazard Rate	Overshoot (Bubble & Crush) Verified by Hazard Rate

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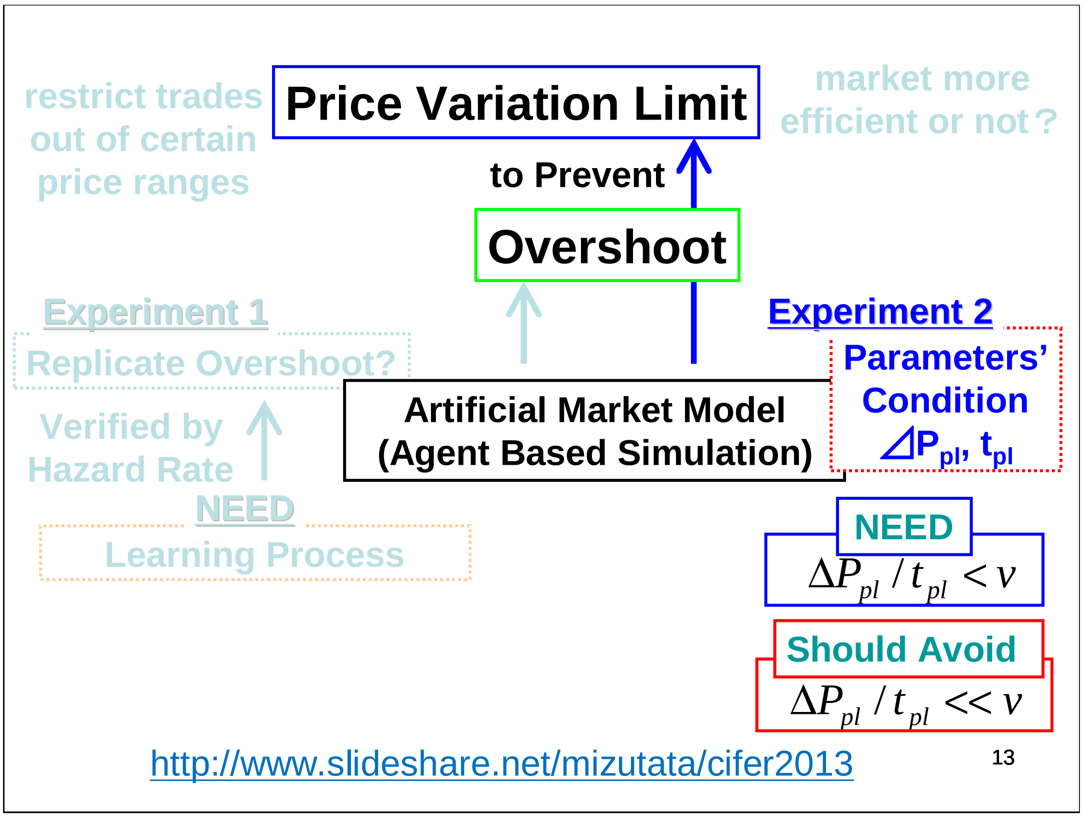
Result Summary Experiment 1 relationship between Learning process and replicating Overshoot

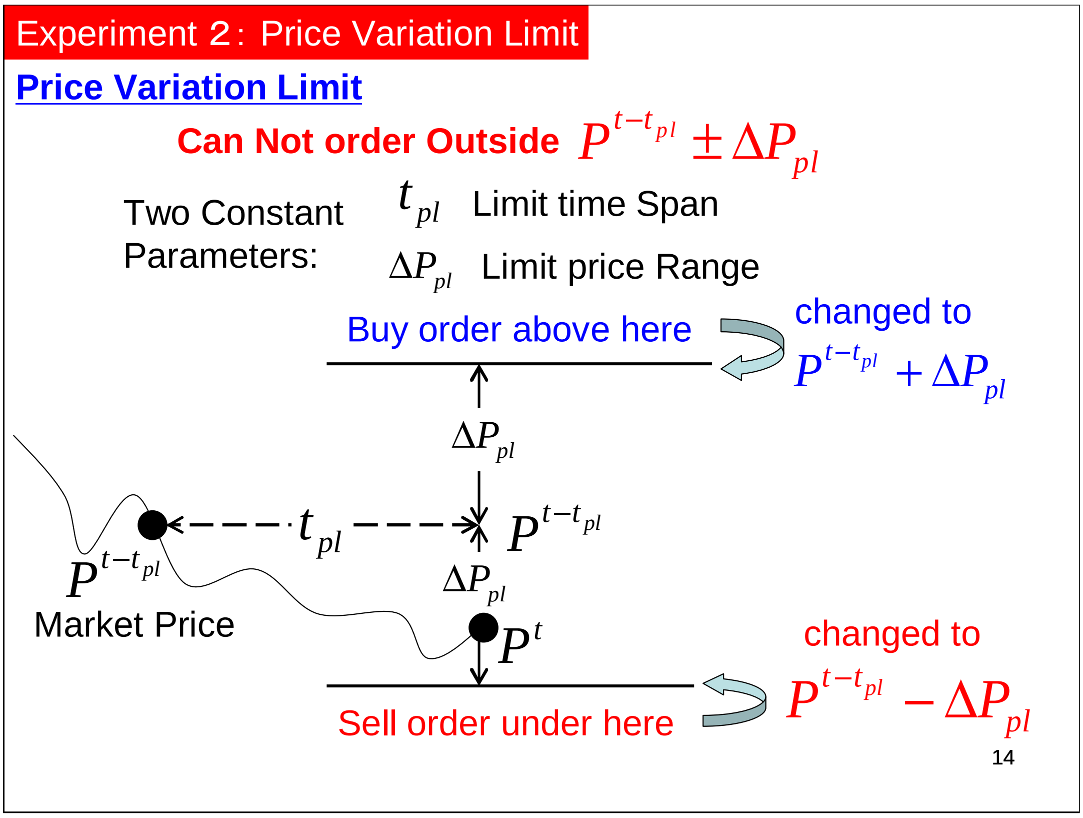
The cases With learning process, both case 1 and case 2, were Consistent with Empirical study verified by Hazard Rate.

And case 2 can replicate overshoot, bubble and crush

The cases Without Learning Process were Not consistent with Empirical study Not verified by Hazard Rate.



Next, I show Simulation Results about Price Variation Limit.



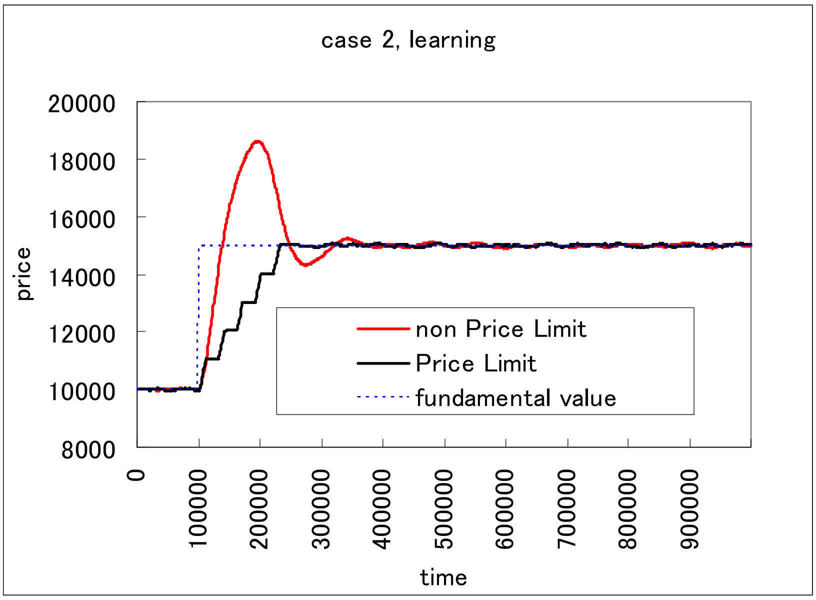
We modeled the price variation limit like this.

There are two constant parameters.
Tpl is a limit time span, and ⊿Ppl is limit price range.

We referred market price Before tpl, Pt-tpl.
and any agents can Not order OutSide from Pt-tpl - ⊿Ppl to Pt-tpl + ⊿Ppl

Concretely,Any buy order prices above here, they are changed to this price.
and any sell order prices under here they are changed to this price.

Case 2 & with Learning, Price Variation Limit



Overshooting was vanished in price variation limit
But, price took longer to reach new fundamental price

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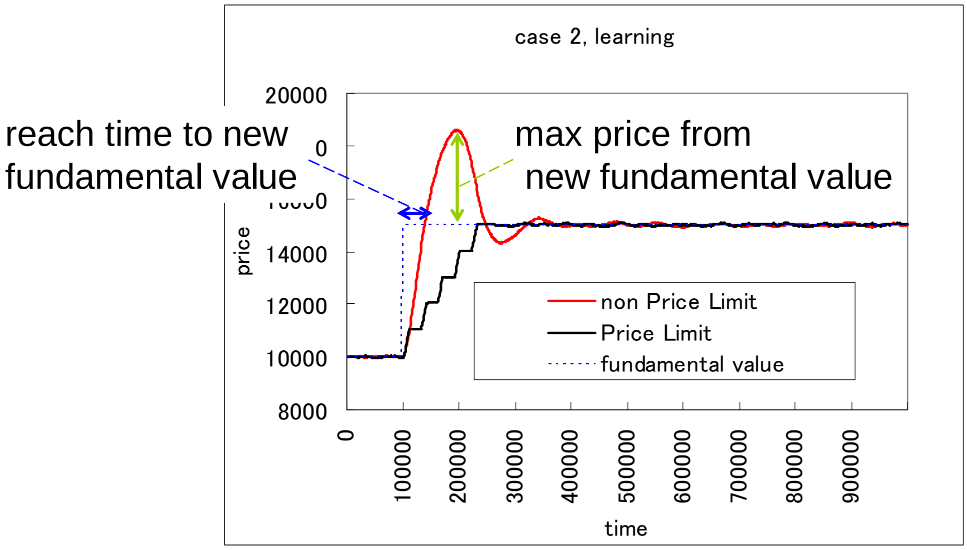
This Figure shows time evolution of prices in case 2 with learning,
comparing the case implemented price variation limit and not implemented.

Overshoot was vanished In the case implemented price variation limit
However, price took longer to reach new fundamental price.

In the case not implemented price variation limit, price took faster to reach new fundamental price.
Implemented case, slower to reach new fundamental value, converging is slower.

Overshooting Amplitude and Converging Speed



Preventing Overshooting and Immediate Converging
→ no market achieves both at once
Optimization of t_{pl} and ΔP_{pl}

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Next, I investigated relationship between Overshooting Amplitude and Converging Speed.

I measured, here, reach time to new fundamental value, and here, max price from new fundamental value.

We want Preventing Overshooting and Immediate Converging.
However, No market achieves both at once.
Therefore, it is important to Optimize of these Two parameters.

Max Price from New Fundamental Value

max price from new fundamental value		t_{pl} 1000	2000	3000	4000	7000	10000	15000	20000	25000	30000	40000	50000
⊿Ppl	100	1,374	475	285	205	115	76	---	---	---	---	---	---
	200	3,236	1,421	728	485	224	146	97	72	---	---	---	---
	300	3,450	3,020	1,378	900	387	233	134	106	90	86	---	---
	400	3,482	3,501	2,381	1,307	565	339	164	112	95	93	85	---
	700	3,347	3,470	3,366	3,311	1,395	734	363	211	120	94	90	83
	1000	3,494	3,357	3,229	3,578	2,633	1,388	681	421	267	122	93	91
	1500	3,512	3,356	3,424	3,404	3,407	3,019	1,384	607	286	208	184	183
	2000	3,454	3,595	3,334	3,317	3,682	3,493	2,408	1,094	580	452	411	418
	2500	3,475	3,357	3,497	3,368	3,262	3,466	3,524	2,007	1,442	508	103	100
	3000	3,436	3,443	3,598	3,431	3,268	3,330	3,365	2,921	1,602	1,190	980	931
	4000	3,359	3,398	3,374	3,607	3,673	3,378	3,338	3,351	3,433	1,701	477	401
	5000	3,556	3,467	3,317	3,509	3,440	3,162	3,486	3,311	3,320	3,231	1,345	104

Blue area: $\frac{\Delta P_{pl}}{t_{pl}} < v \cong 0.128$

v: Converging Speed without price variation limit

Preventing Overshooting

$\Delta P_{pl} / t_{pl}$ smaller (to upper right) ⇒ Overshoot smaller

NEED to prevent overshoots $\Delta P_{pl} / t_{pl} < v$

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This table lists Max Price from New Fundamental Value in various Tpl and ⊿Ppl

Blue area, ⊿Ppl over Tpl is smaller than V

V is a converging speed without price variation limit, approximately 0.128.

As you see, in this area, max price small, this means preventing overshoot.

This is smaller, to upper right area, overshoot are smaller.

Therefore, we found that this is smaller than V is needed to prevent overshoot.

Reach time to New Fundamental Value

reach time to new fundamental value (x 1000)		t_{pl}											
		1000	2000	3000	4000	7000	10000	15000	20000	25000	30000	40000	50000
⊿Ppl	100	55	104	157	216	382	555	---	---	---	---	---	---
	200	40	55	79	103	181	261	385	509	---	---	---	---
	300	39	41	55	70	120	171	257	342	425	506	---	---
	400	39	39	44	55	91	128	195	259	324	385	505	---
	700	40	39	40	39	55	75	113	148	185	229	302	371
	1000	39	39	39	39	41	54	78	104	127	150	193	234
	1500	39	39	39	39	39	40	54	72	93	113	146	175
	2000	39	39	40	40	39	39	43	56	71	85	106	126
	2500	39	39	39	39	39	39	39	46	53	60	74	86
	3000	39	39	39	39	40	39	39	40	48	56	70	79
	4000	39	39	39	39	39	39	40	39	39	44	63	76
	5000	39	39	39	39	39	40	39	39	40	40	40	47

Blue area: $\frac{\Delta P_{pl}}{t_{pl}} < v \cong 0.128$ v: Converging Speed without price variation limit

$\Delta P_{pl} / t_{pl}$ smaller (to upper right) ⇒ converging speed slower

Should Avoid Not to be converging slower $\Delta P_{pl} / t_{pl} << v$

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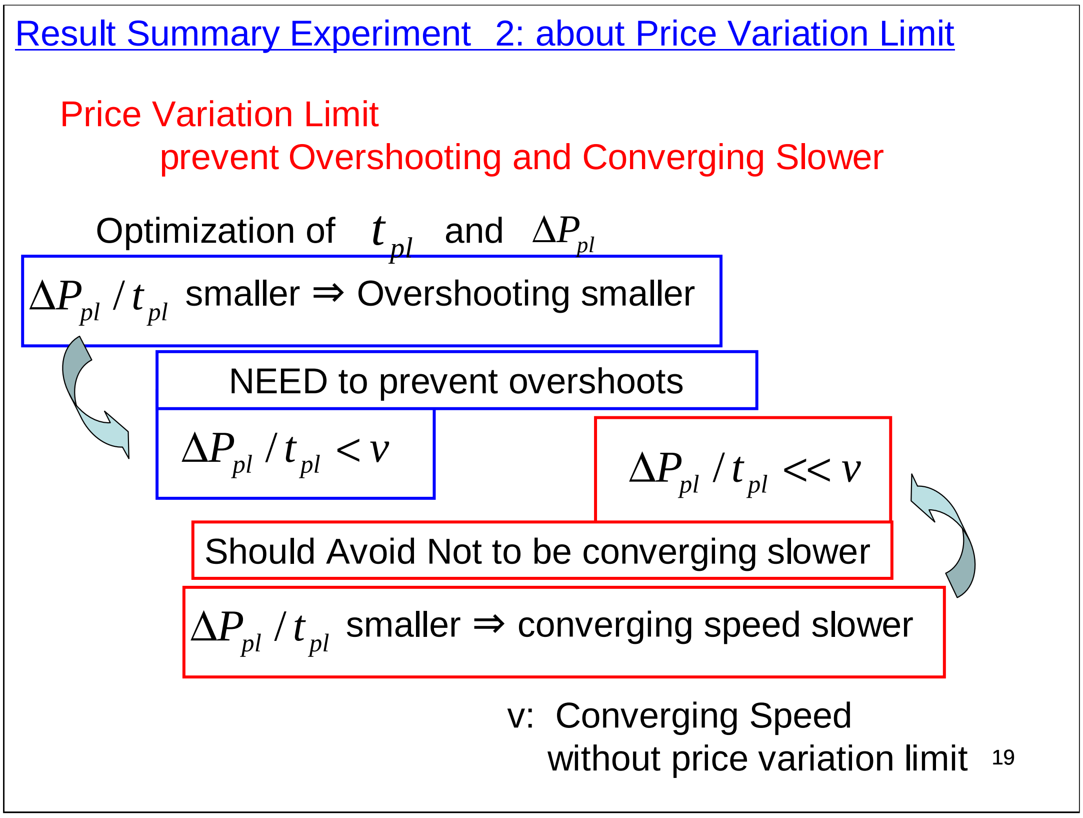
This table lists Reach time to New Fundamental Value in various Tpl and ⊿Ppl

Blue area, this is smaller than V.

This is smaller, to upper right area, converging speed are slower.
Therefore, we found that it this is too smaller than V is should be Avoid Not to be converging slower.

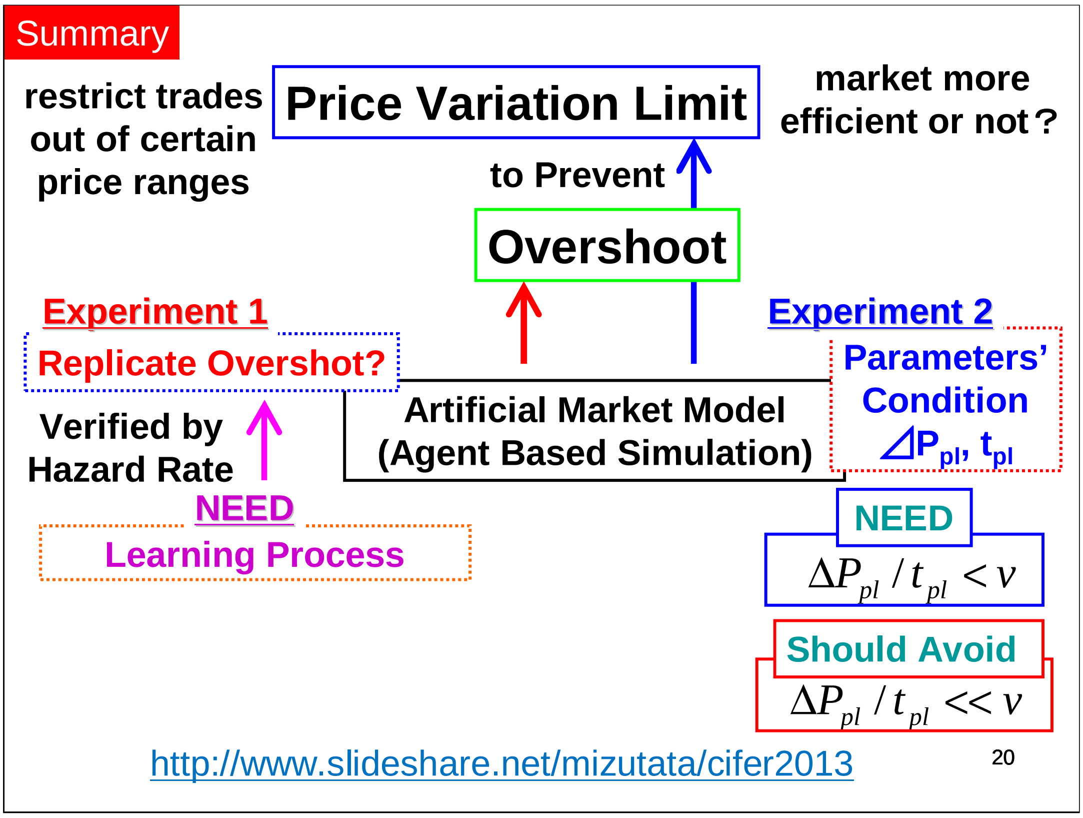


Result Summary Experiment 2 about Price Variation Limit.

Price Variation Limit prevents Overshoot and cause Converging speed Slower to new fundamental value
Therefore, it is important to Optimize of these two parameters.

When this is smaller, overshooting smaller.
We found that it needs smaller than, V to prevent overshoots

On the other hand, this is smaller, converging speed is slower.
We found that it should be avoid too smaller than V Not to be converging slower



I summarize this presentation

■ To investigate price variation limits, we built an artificial market model implementing a learning process.

■ Experiment 1, we showed that the model should be implemented the learning process to replicate overshoot.

■ Experiment 2, We also showed that a parameters' condition of the price variation limit to prevent overshoot.

That’s all for my presentation.

Thank you very much
for your cooperation !

http://www.slideshare.net/mizutata/cifer2013

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Could you say that again? (もう一度、おっしゃっていただけますか？)

I don’t quite understand your question. (ご質問の趣旨が良く分からないのですが)

Could you please rephrase your question? (ご質問を分かりやすく言い換えていただけますか)

So, you are asking me about.... (つまり、お尋ねの内容は...ですね)

I totally agree with you. (私も全くあなたと同意見です)

That’s a very challenging question for me to answer. (それは私にとって非常に答えがいのある質問です)

That’s a question I’m not sure I can answer right now. (そのご質問にすぐお答えできるかどうか分かりません)

It would require further research. (さらなる研究結果を待ちたい)

You are right on that point. (その点に関してはあなたが正しい)

Our method will not solve the problem. (我々の方法ではその問題は解決できない)

Appendix

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Artificial Market Model (Agent Based Model)

On basis of Chiarella et. al. [2009]

\+ Learning Process of agents

comparing between the case with Learning Process and without

Feature of our model

○ agent model is Simple

→ to avoid arbitrary result "Keep it short and simple"

Models should explain stylized facts as simply as possible

● pricing mechanism is Continuous Double Auction

→ not simple to implement realistic price variation limit

☆ Learning process

→ agents switch strategy, fundamental or technical

An overshoot occurred in the case with the learning process but did not occur in the case without the process

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We built an artificial market model on basis of Chiarella et. al. [2009].

Chiarella's model did not include Learning Process, however,

We built Learning Process of agents.

And we are comparing between the case with Learning Process and without it.

Our Agent Model is Simple. This is to avoid arbitrary result, "Keep it short and simple" principle.

We think Artificial Market Models should explain Stylized Facts as Simply as possible,

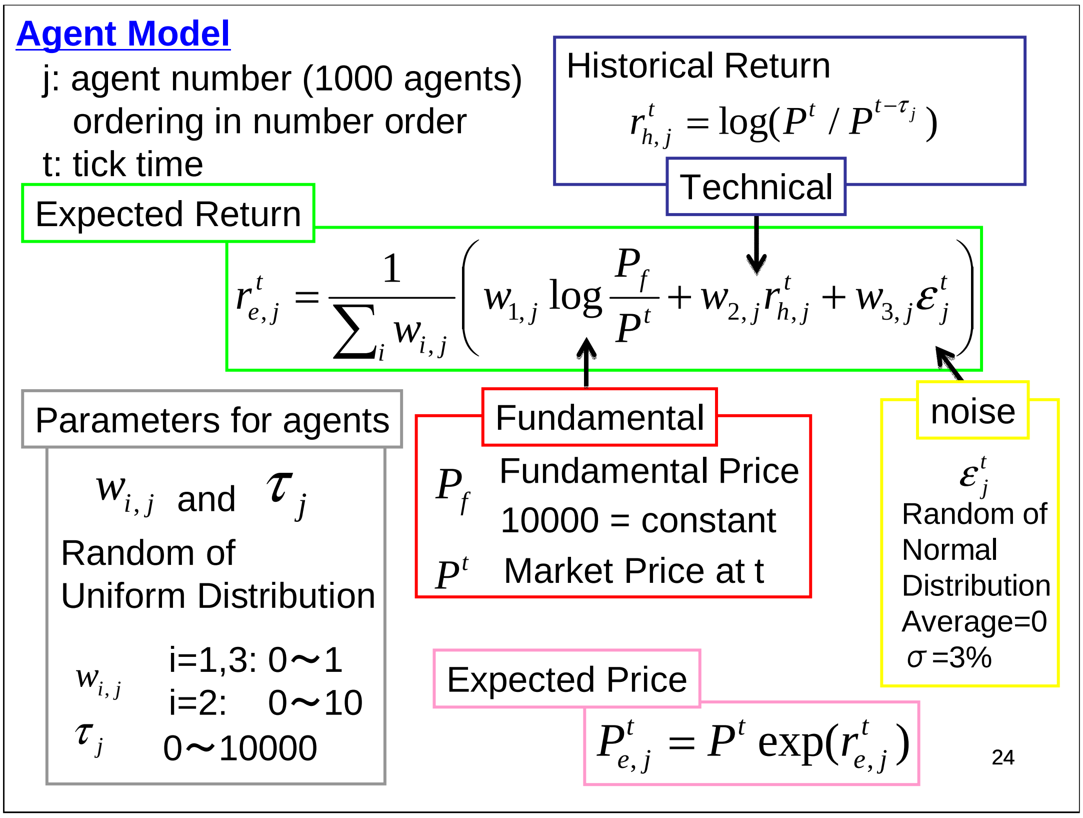
Our pricing mechanism is Continuous Double Auction

It is not simple, but, we need to implement realistic price variation limit

Learning process

Here, Learning process means agents are switching strategy, fundamental strategy or technical strategy.

We will show that, an Overshoot occurred in the case With the learning process, however, overshoot did not occur in the case WithOut the process



Next, I will describe agent model.

All agents calculate Expected Return using this equation.

First term is a Fundamental Strategy:

When the market price is smaller than the fundamental price, an agent expects a positive return , and vice verse.

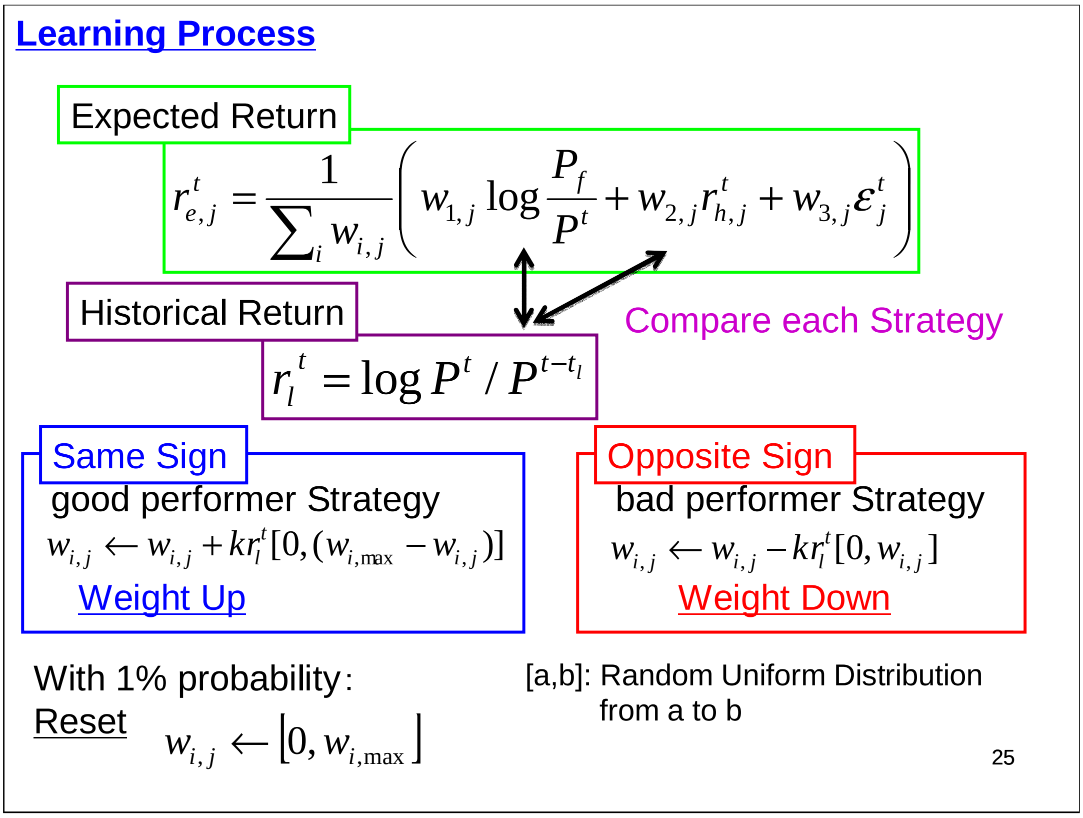
Second term is a technical strategy:

When historical return is positive, an agent expects a positive return, and vice verse.

Third term is noise,

After the expected return has been determined, an expected price is determined like this.

And, agents order base on this Expected Price.



We also developed a model implementing a learning process of agents.

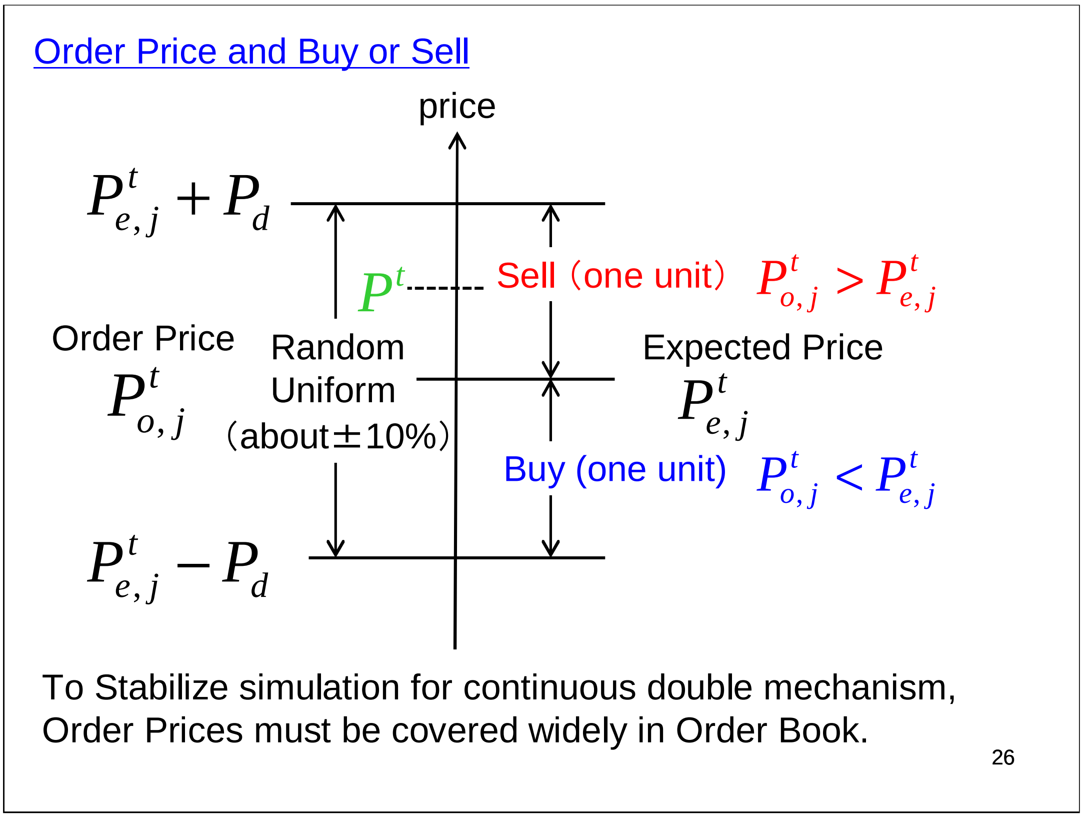
Agents are comparing Historical Return and each Strategy' term, Fundamental strategy term, and Technical strategy term.

When the strategy's expected return and Historical Return are Same Sign,
This means good performer Strategy.
The strategy's Weight is Up.

When the strategy's expected return and Historical Return are Opposite Sign,
This means bad performer Strategy.
The strategy's Weight is Down.

We also added random learning.

In this way, agents learn better parameters and switch to the investment strategy that estimates correctly.



Next, agents determine order price and, buy or sell.

To Stabilize simulation runs for the continuous double mechanism,
Order Prices must be covered widely in Order Book.

We modeled an Order Price, Po, by Random variables of Uniformly distributed in the interval from Expected Price, Pe, minus constant, Pd, to Pe plus Pd.

And then,
When Po lager than Pe, the agent orders to sell one unit.
When Po smaller than Pe, the agent orders to buy one unit.

Hazard Rate

		case 2 lerning	
		non Price Limit	Price Limit
	i		
hazard rate	1	55%	55%
	2	50%	53%
	3	45%	49%
	4	40%	47%
	5	36%	44%
	6	29%	42%
	7	26%	40%
	8	22%	36%
	9	19%	31%

Hazard Rates increased
→ Result by preventing bubble

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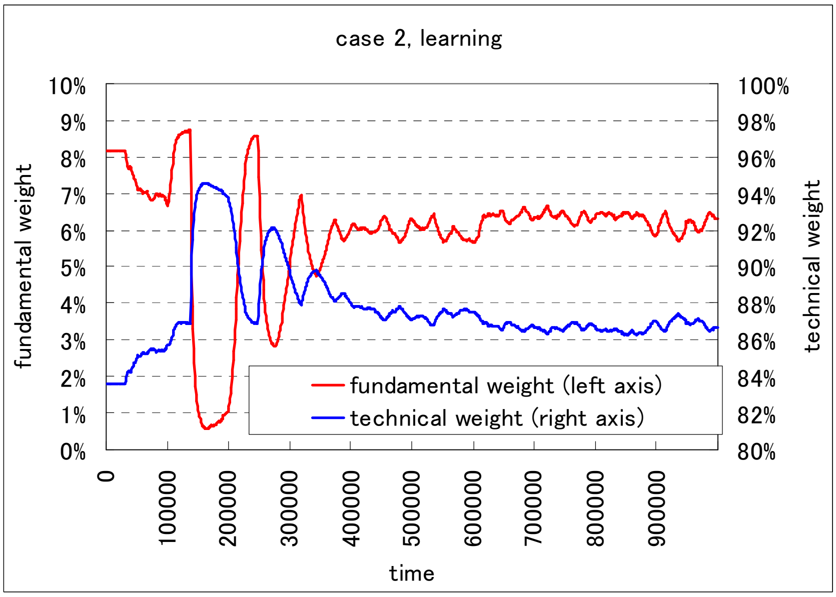
Hazard Rate

Hazard Rates increased in implemented price variation limit
This Result shows preventing bubble

Strategies Weight



During overshooting, switching fundamental to technical
Consistent with empirical studies

[Frankel and Froot, 1990], [Yamamoto and Hirata, 2012]

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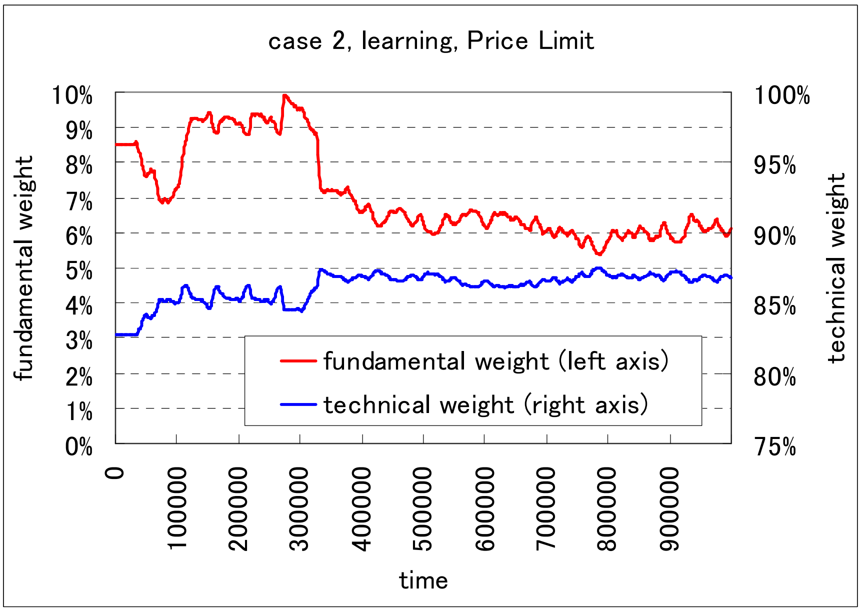
I also show Strategies weight, in the case 2 with learning
which case include overshoot.

During overshooting, agents are switching fundamental strategy to technical strategy.

This is consistent with empirical studies, like these studies.

Strategies Weight



Agents Not switching to the technical strategy
→ prevented overshooting.

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I also show Strategies weight, in the case 2 with learning, implemented price variation limit

Agents are not switching fundamental strategy to technical strategy.

This causes preventing overshooting, bubble and crash.